Proving Preservation of Partial Correctness with ACL2: A Mechanical Compiler Source Level Correctness Proof

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Outline:

- ➜ Background, Three Steps to Correct Realistic Compilation
- ➜ Source Level Verification is not Sufficient
- ➜ Correct Implementation, Preservation of Partial Correctness
- ➜ Source and Target Language, the Compiler
- ➜ The Correctness Proof in ACL2
- ➜ Conclusions and Further Work

Generate correct executables from correct source programs

- manually
- using unverified compilers

- using verified compilers (trusted compiler executables)

  Verifix DFG research group (Karlsruhe, Kiel, Ulm)

  for realistic source languages and real target processors

Generate correct executables from correct source programs

- manually
- using unverified compilers

  without verified compiling specification

  - manually semantically checked [state-of-the-art certification]
  - semantically checked by machine [Pnueli et al., Necula 1998, translation validation]

  with verified compiling specification

  - manually syntactically checked [Goerigk,Hoffmann 1998]
  - syntactically checked by machine [Traverso et al., 1998]

- using verified compilers (trusted compiler executables)

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Verifix Goals

Construct and correctly implement compilers and compiler generators

- ➜ for realistic imperative and object-oriented source languages
- ➜ for real target and host processors
- ➜ generating efficient code that compares to unverified compilers
- ➜ exploiting mechanical proof support, e.g., by PVS or ACL2
- ➜ industrially approved compiler architecture and construction techniques
- ➜ proof methodology supplements compiler construction, not vice versa

- ➜ exploit runtime result verification
  (a posteriori program or result checking) and
- ➜ an initial fully trusted compiler as sound bootstrapping basis

① Specification of a compiling relation $\mathcal{C}_{\mathbf{TL}}^{\mathbf{SL}}$ between abstract source and target languages **SL** and **TL**, and compiling (specification) verification w.r.t. language semantics $[\![\cdot]\!]_{\mathbf{SL}}$, $[\![\cdot]\!]_{\mathbf{TL}}$ and an appropriate semantics relation $\sigma_{\mathbf{TL}}^{\mathbf{SL}}$.

② Implementation of a corresponding compiler program $\pi_{\mathbf{SL}}$ in high level implementation language **SL** (close to the specification language), and high level compiler implementation verification w.r.t. $\mathcal{C}_{\mathbf{TL}}^{\mathbf{SL}}$.

③ Low level implementation of a corresponding compiler executable $m_{\mathbf{TL}}$ written in binary target machine language **TL**, and low level compiler implementation verification w.r.t. $[\![\pi_{\mathbf{SL}}]\!]_{\mathbf{SL}}$.

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theoretical comp. sc., progr. lang. theory, [McCarthy and Painter 1967], ...

② Implementation of a corresponding compiler program $\pi_{\mathbf{SL}}$ in high level implementation language **SL** (close to the specification language), and high level compiler implementation verification w.r.t. $\mathcal{C}_{\mathbf{TL}}^{\mathbf{SL}}$.
[Polak 1981], [Moore 1988, 1996], [Curzon 1994, 1996]
software eng., formal methods like VDM, RAISE, CIP, PROSPECTRA, Z, B, ...

③ Low level implementation of a corresponding compiler executable $m_{\mathbf{TL}}$ written in binary target machine language **TL**, and low level compiler implementation verification w.r.t. $[\![\pi_{\mathbf{SL}}]\!]_{\mathbf{SL}}$.
virtually nothing, only demands [Chirica and Martin 1986], [Moore 1988]

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③′ Strong Compiler Bootstrap Test: Compile $\pi_{\mathbf{SL}}$ to $m_{\mathbf{TL}}$ by a twofold bootstrapping, using an unverified **SL**-compiler $\overline{m}$. Apply $m_{\mathbf{TL}}$ to $\pi_{\mathbf{SL}}$ and test if $m_{\mathbf{TL}}$ reproduces itself.

DEMO

Reasonable Semantical Relations

Semantical relations $\sigma_{\mathsf{TL}}^{\mathsf{SL}} : \mathrm{Sem}_{\mathsf{SL}} \rightharpoonup \mathrm{Sem}_{\mathsf{TL}}$ express notions of correct implementation. Here are some wishes:

- ➜ handle non-determinism of the source program semantics
- ➜ handle resource limitations of the target machine
- ➜ allow for optimizations that require well-definedness properties of the source program
- ➜ handle (non-terminating) reactive programs, e.g., preserve definedness properties of the source program
- ➜ allow for full recursion and dynamic data types, e.g. for transformational programs like compilers, ...

Why Non-Determinism - An Example

```
procedure p ();
  begin int x; x := 42 end;

procedure q ();
  begin int y; print (y) end;

begin p(); q() end.
```

Specification Refinement (intuitive):

The implementation should at least return every specified result, i.e., it should be at least as defined as the specification.

Preservation of Partial Correctness (intuitive):

The implementation should at most return specified results, i.e., we do not want to see any non-erroneous incorrect result.

Ω : error outcomes, $A \subseteq \Omega$ acceptable errors, $U = \Omega \setminus A$ unacceptable (chaotic) errors

$$
\begin{tikzcd}
[\![\, \pi \,]\!]_{\mathsf{SL}} \in \mathsf{Sem}_{\mathsf{SL}} \quad : & {}_i D^{\Omega}_{\mathsf{SL}} \arrow[r, "{[\![\, \pi \,]\!]_{\mathsf{SL}}}"] \arrow[d, "{}_i\rho^{\mathsf{SL}}_{\mathsf{TL}}"'] \arrow[dr, phantom, "\Nearrow A"] & {}_o D^{\Omega}_{\mathsf{SL}} \arrow[d, "{}_o\rho^{\mathsf{SL}}_{\mathsf{TL}}"] \\
[\![\, m \,]\!]_{\mathsf{TL}} \in \mathsf{Sem}_{\mathsf{TL}} \quad : & {}_i D^{\Omega}_{\mathsf{TL}} \arrow[r, "{[\![\, m \,]\!]_{\mathsf{TL}}}"'] & {}_o D^{\Omega}_{\mathsf{TL}}
\end{tikzcd}
$$

$\sigma^{\mathsf{SL}}_{\mathsf{TL}}$

Definition: m *correctly implements* π *relative to* A, iff for any $d \in {}_i D^{\Omega}_{\mathbf{SL}}$ with $(\, [\![\, \pi \,]\!]_{\mathsf{SL}} \,;\, {}_o\rho^{\mathsf{SL}}_{\mathsf{TL}})\,(d) \cap U = \emptyset$ we have

$$(\, {}_i\rho^{\mathsf{SL}}_{\mathsf{TL}} \,;\, [\![\, m \,]\!]_{\mathsf{TL}} \,)\,(d) \subseteq (\, [\![\, \pi \,]\!]_{\mathsf{SL}} \,;\, {}_o\rho^{\mathsf{SL}}_{\mathsf{TL}})\,(d) \cup A$$

[Goerigk/Langmaack 2000], [Müller-Olm/Wolf 1999]

Choose $\Omega =_{\text{def}} \{\bot\}$ and $A =_{\text{def}} \{\bot\}$ $\quad [\Longrightarrow U = \Omega \setminus A = \emptyset\,]$.

$$
\begin{tikzcd}
[\![\,\pi\,]\!]_{\mathsf{SL}} \in \mathsf{Sem}_{\mathsf{SL}} \quad : & {}_{i}D_{\mathsf{SL}}^{\{\bot\}} \arrow[r, "{[\![\,\pi\,]\!]_{\mathsf{SL}}}"] \arrow[d, "{}_{i}\rho_{\mathsf{TL}}^{\mathsf{SL}}"'] \arrow[dr, phantom, "\Downarrow"] & {}_{o}D_{\mathsf{SL}}^{\{\bot\}} \arrow[d, "{}_{o}\rho_{\mathsf{TL}}^{\mathsf{SL}}"] \\
[\![\,m\,]\!]_{\mathsf{TL}} \in \mathsf{Sem}_{\mathsf{TL}} \quad : & {}_{i}D_{\mathsf{TL}}^{\{\bot\}} \arrow[r, "{[\![\,m\,]\!]_{\mathsf{TL}}}"'] & {}_{o}D_{\mathsf{TL}}^{\{\bot\}}
\end{tikzcd}
$$

$\sigma_{\mathsf{TL}}^{\mathsf{SL}}$

Definition: We say that m *L-simulates* π (or that the step $\pi \mapsto m$ *preserves partial correctness*) iff

$$\left({}_{i}\rho_{\mathsf{TL}}^{\mathsf{SL}} \,;\, [\![\,m\,]\!]_{\mathsf{TL}} \right) \subseteq \left([\![\,\pi\,]\!]_{\mathsf{SL}} \,;\, {}_{o}\rho_{\mathsf{TL}}^{\mathsf{SL}} \right)$$

[Goerigk et al. 1996], [Müller-Olm 1996]

Syntax:

$$
\begin{array}{lcl}
p & ::= & (\texttt{(}d_1 \ \ldots \ d_n\texttt{)}\texttt{(}x_1 \ \ldots \ x_k\texttt{)}\ e\texttt{)} \\
d & ::= & \texttt{(defun}\ f\ \texttt{(}x_1 \ \ldots \ x_n\texttt{)}\ e\texttt{)} \\
e & ::= & c \mid x \mid \texttt{(if}\ e_1\ e_2\ e_3\texttt{)} \mid \texttt{(}f\ e_1 \ \ldots \ e_n\texttt{)} \mid \texttt{(}\mathit{op}\ e_1 \ \ldots \ e_n\texttt{)}
\end{array}
$$

A Sample Program - Factorial:

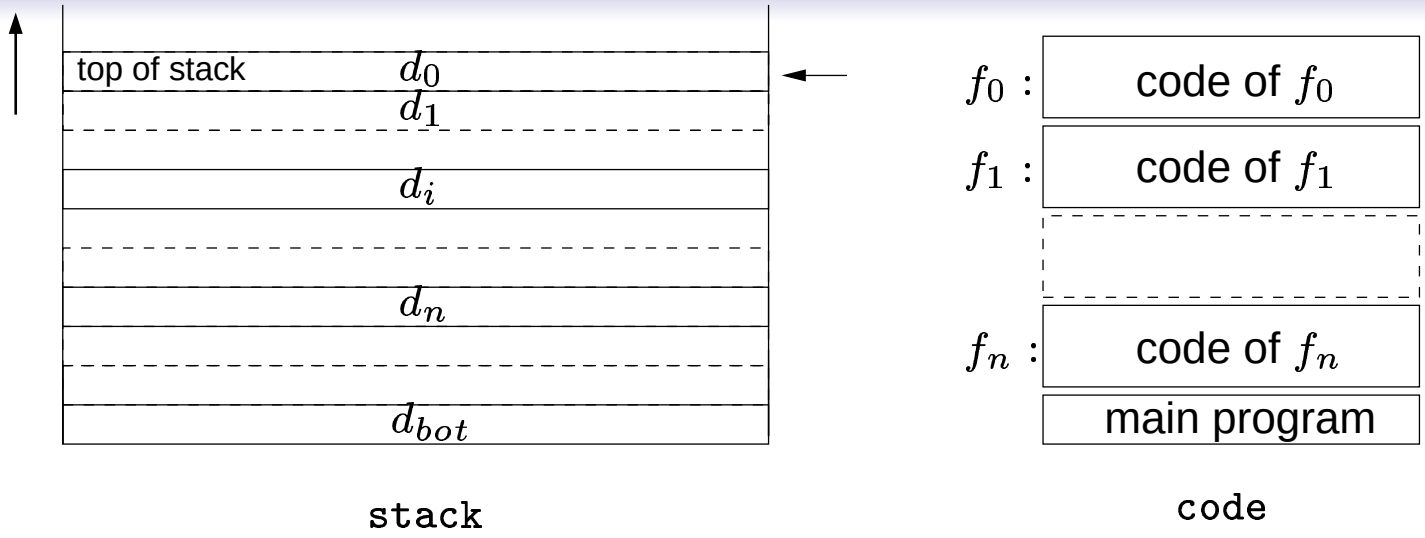
```
(((defun fac (n) (if (= n 0) 1 (* n (fac (1- n))))))
 (n)
 (fac n))
```

Operational Semantics (interpreter function):

```
(defun evaluate (defs vars main inputs n) ...)
```

Semantics of forms (expressions):

```
(defun evl (form genv env n) ...)   returns ([[form]]) or  error
(defun evlist (forms genv env n) ...)
```



Machine Instructions

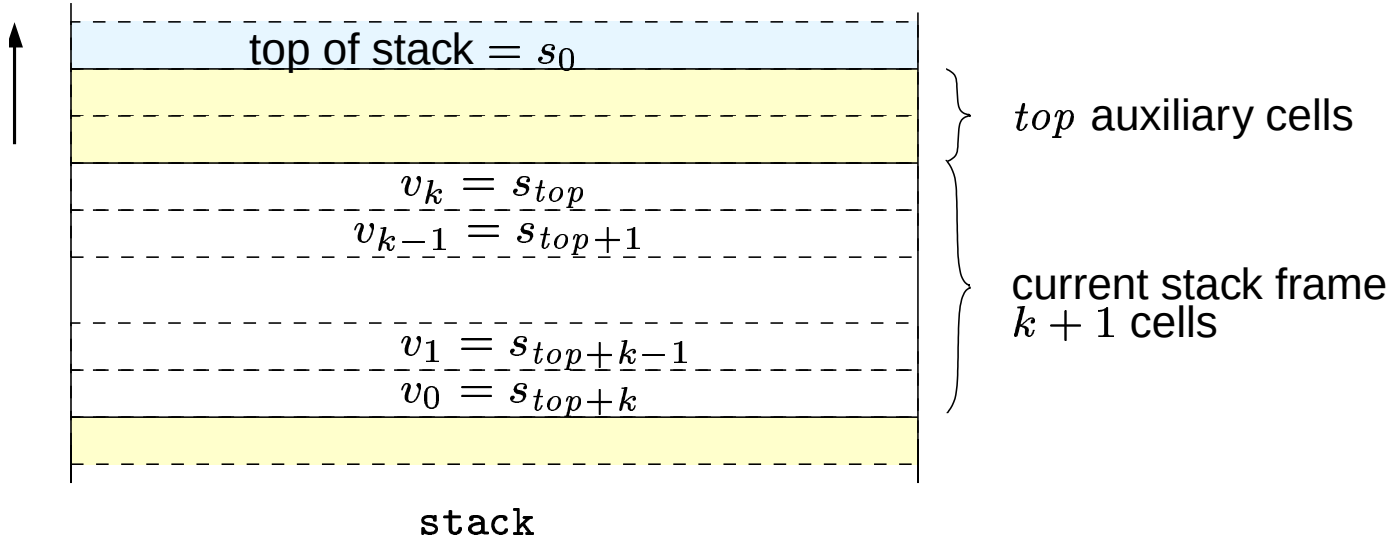
(PUSHC c) (PUSHV i) (POP n) (IF m_1 m_2) (OPR op) (CALL f)

Operational Semantics (interpreter function):

```
(defun execute (prog stack n) ...)
```

Stepwise Execution of Machine Instructions:

```
(defun mstep (instr code stack n) ... )
(defun msteps (instr-seq code stack n) ... )
```

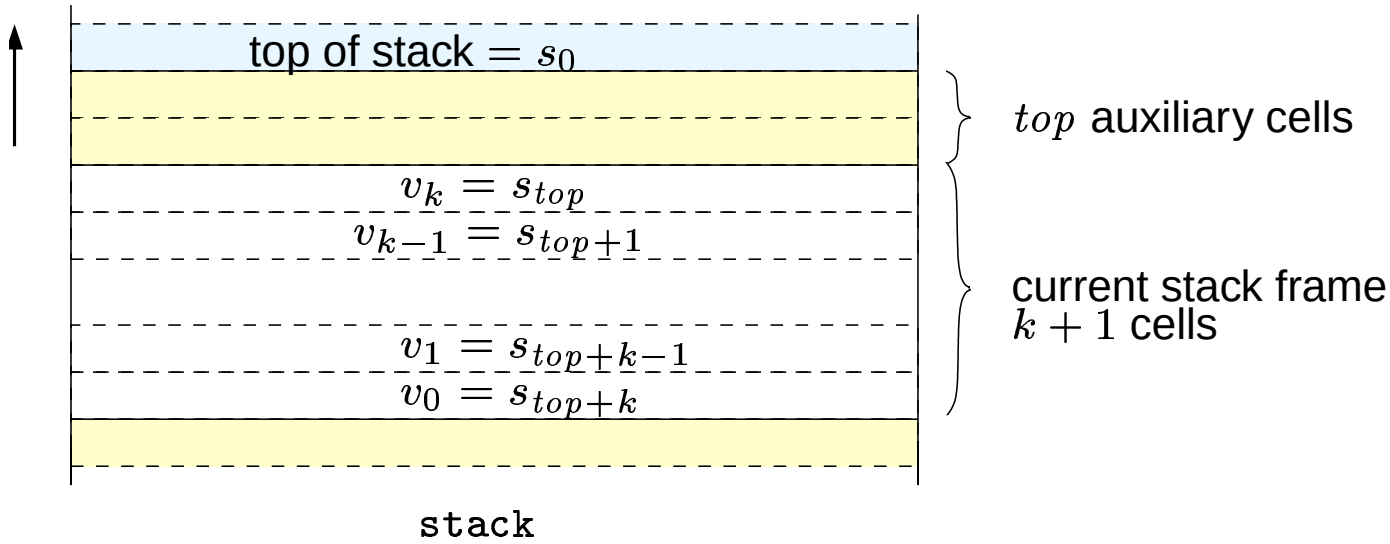


We compile expressions according to the stack principle:

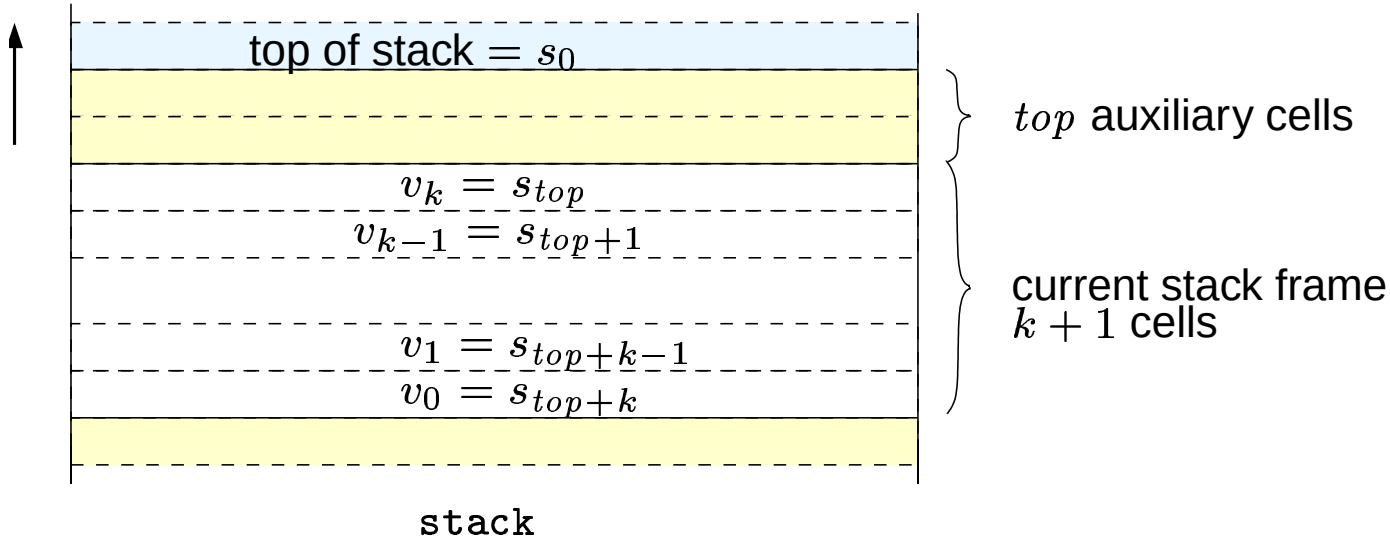
The instruction sequence m for the expression e pushes the value v of e onto the stack. Operators and functions consume their arguments.

Variable Access

For any x_i in $(x_0 \ \ldots \ x_k)$ we find the value of x_i at position $top + |x_i \ \ldots \ x_k| - 1$ on the stack.



$$
\begin{aligned}
\mathit{compile\text{–}form}\ (\mathit{form}, (x_0\ \ldots\ x_k), \mathit{top}) &= \mathit{form}'_{top} = \\
c &\mapsto ((\texttt{PUSHC c})) \\
x_i &\mapsto ((\texttt{PUSHV}\ \mathit{top} + |x_i\ \ldots\ x_k| - 1)) \\
(\texttt{if}\ e_1\ e_2\ e_3) &\mapsto e'_{1,top} \cdot (\texttt{IF}\ e'_{2,top}\ e'_{3,top}) \\
(f\ e_0\ \ldots\ e_n) &\mapsto e'_{0,top} \cdot \ldots \cdot e'_{n,top+n} \cdot (\texttt{CALL}\ f) \\
(op\ e_0\ \ldots\ e_n) &\mapsto e'_{0,top} \cdot \ldots \cdot e'_{n,top+n} \cdot (\texttt{OPR}\ op)
\end{aligned}
$$

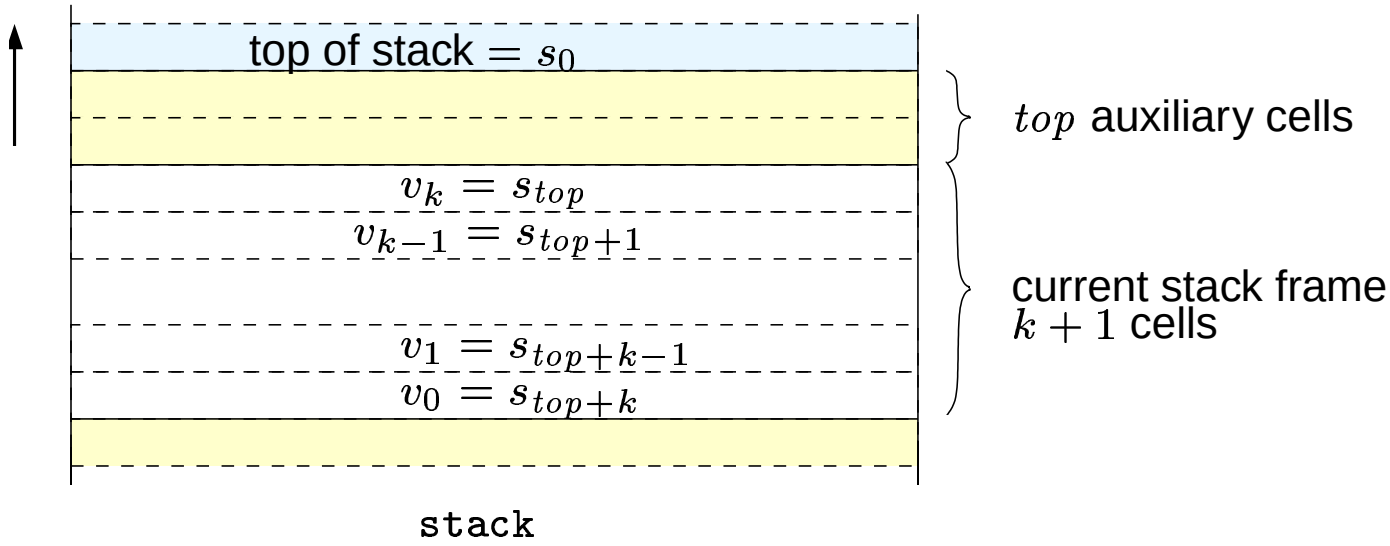


$$\begin{aligned} \texttt{env} &= (\texttt{bind}\ (x_0 \ldots x_k)\ (\texttt{rev}\ (\texttt{get-stack-frame}\ (x_0\ \ldots\ x_k)\ top\ s))) \\ &= ((x_0\ .\ s_{top+k})\ \ldots\ (x_k\ .\ s_{top})) \end{aligned}$$

Lemma 1 (Variable access). For any $\texttt{n} \geq 1$, $(\texttt{evl}\ x_i\ \texttt{genv}\ \texttt{env}\ \texttt{n})$ is defined and

$$\underbrace{s_{top+k-i} \cdot s}\ = (\texttt{car}\ (\texttt{evl}\ x_i\ \texttt{genv}\ \texttt{env}\ \texttt{n})) \cdot s$$

$$= (\texttt{mstep}\ (\texttt{PUSHV}\ top + |x_i\ \ldots\ x_k| - 1)\ \ldots\ s\ \texttt{n})$$
$$= (\texttt{msteps}\ (\texttt{compile-form}\ x_i\ (x_0\ \ldots\ x_k)\ top)\ \ldots\ s\ \texttt{n})$$



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Lemma 2 (Constants). For any $\texttt{n} \geq 1$, $(\texttt{evl}\ c\ \texttt{genv}\ \texttt{env}\ \texttt{n})$ is defined and

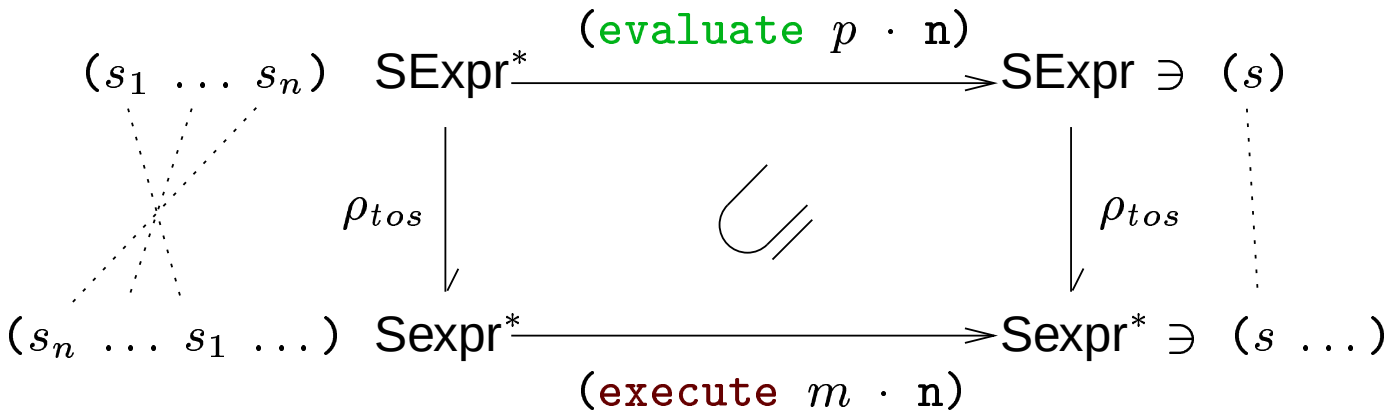
$$\underbrace{c \cdot s} = (\texttt{car}\ (\texttt{evl}\ c\ \texttt{genv}\ \texttt{env}\ \texttt{n})) \cdot s$$

$$= (\texttt{mstep}\ (\texttt{PUSHC}\ c)\ \ldots\ s\ \texttt{n})$$
$$= (\texttt{msteps}\ (\texttt{compile-form}\ c\ (x_0 \ldots x_k)\ top)\ \ldots\ s\ \texttt{n})$$

Theorems 1 and 2 (Compiler correctness for forms (form lists))

If the machine, executed on a compiled `form` (list), is defined on a `stack` for an `n`, then the following three conjectures hold:

1. The semantics of the `form` (list) – in the given function environment and with the free variables bound to their values in the current stack-frame – is defined for the same `n`.
2. The machine returns a new stack with the value(s) of the `form`(s) on top (in reverse order).
3. The stack just below the result value(s) remains unchanged.



Theorem 3 (Compiler preserves partial correctness)

```
(defthm compiler-correctness-for-programs
  (let ((new-stack (execute (compile-program defs vars main)
                            (append (rev inputs) stack) n))
        (value (car (evaluate defs vars main inputs n))))
    (implies
      (and (wellformed-program defs vars main) (defined new-stack)
           (true-listp inputs) (equal (len vars) (len inputs)))
      (equal new-stack (cons value stack)))))
```

Theorem 1 (Compiler correctness for forms)

```
(defthm compiler-correctness-for-forms
  (let ((value
          (evl    form
               (construct-genv dcls)
               (bind cenv (rev (get-stack-frame cenv top stack)) env)
               n))
        (new-stack (msteps (compile-form  form  cenv top)
                           (download (compile-defs dcls)) stack n)))
  (implies
    (and (natp top)
         (wellformed-defs dcls (construct-genv dcls))
         (wellformed-form  form  (construct-genv dcls) cenv)
         (defined new-stack))
    (and (defined value )
         (equal new-stack (cons (car value) stack))))))
```

Theorem 2 (Compiler correctness for form lists)

```
(defthm compiler-correctness-for-form-lists
  (let ((values
          (evlist forms
                  (construct-genv dcls)
                  (bind cenv (rev (get-stack-frame cenv top stack)) env)
                  n))
        (new-stack (msteps (compile-forms forms cenv top)
                           (download (compile-defs dcls)) stack n)))
  (implies
    (and (natp top)
         (wellformed-defs dcls (construct-genv dcls))
         (wellformed-forms forms (construct-genv dcls) cenv)
         (defined new-stack))
    (and (defined values)
         (equal new-stack (append (rev values) stack))))))
```

Induction on n and the structural depth of forms

```
(defun compiler-induction (flag x cenv env top dcls stack n)
  (declare (xargs :measure (cons (1+ (acl2-count n)) (acl2-count x))))
  (if (or (zp n) (atom x)) (list x cenv env top dcls stack n)
   ...
    ;; function call
    (list (compiler-induction nil
            (cdr x) cenv env top dcls stack n)
          (compiler-induction t
            (get-body (car x) (construct-genv dcls))
            (get-vars (car x) (construct-genv dcls))
            (bind cenv (rev (get-stack-frame cenv top stack)) env)
            0 dcls
            (msteps (compile-forms (cdr x) cenv top)
                    (download (compile-defs dcls))
                    stack n)
            (1- n))))))
   ...) ...)
```

Prove Theorems 1 and 2 simultaneously:

```
(defmacro theorem-1 (form cenv env top dcls stack n) ...)
(defmacro theorem-2 (forms cenv env top dcls stack n) ...)
```

```
(defthm compiler-correctness-form-forms
  (if flag
      (theorem-1 x cenv env top dcls stack n)
    (theorem-2 x cenv env top dcls stack n))
  :hints (("Goal"
           :induct (compiler-induction flag x cenv env top dcls stack n)
           ...)))

(defthm compiler-correctness-for-expressions
  (theorem-1 x cenv env top dcls stack n)
  :hints (("Goal" :by
            (:instance compiler-correctness-form-forms (flag t)))))
```

Conclusions

- ➜ We seriously and rigorously have to tackle target level implementation verification as well
- ➜ Source level verification and testing or validation alone are not sufficient!
- ➜ As it stands, this fact is now mechanically proved in ACL2. [Goerigk 1999, 2000].
- ➜ There is a repeatable technique for constructing initial, fully verified compiler implementations from the scratch and for realistic systems implementation languages [Goerigk and Hoffmann 1998, Hoffmann 1998] $\mapsto$ a major Goal of *Verifix*
- ➜ The known gap between high level verification and software integration [Verifix, since 1994, BSI, 1996] can be closed

Some Future Work

- ➜ Formalize further compilation phases, i.e., data refinement, code linearization, machine code generation
- ➜ Prove full compiler correctness formally and mechanically in ACL2 (including target level implementation correctness)